Chapter 11

Useful formulae: $2\pi f = \omega = 2\pi/T$. $v_{max} = \omega A$. $v^2/v_{max}^2 + x^2/A^2 = 1$.

1. $F = Kx$
$-22\ \text{N} = -k(0.35\ \text{m} - l)$, $-60\ \text{N} = -k(0.58\ \text{m} - l)$
$-22\ \text{N} = -0.35\ \text{m}k + kl$, $-60\ \text{N} = -0.58\ \text{m}k + kl$
$-38\ \text{N} = -0.23\ \text{m}k$
$k = \underline{165\ \text{N/m}}$

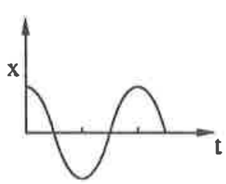
2. $k = -(70\ \text{kg})(9.8\ \text{m/s}^2)/(-0.022\ \text{m}) = 3.12 \times 10^4\ \text{N/m}$

$f = (\text{k/m})^{\frac{1}{2}}/2\pi = [(3.12 \times 10^4\ \text{N/m})/1270\ \text{kg}]^{\frac{1}{2}}/2\pi = \underline{0.789\ \text{Hz}}$

3. In one period the particle goes out A, comes back, goes out other way, comes back. Total distance is $\underline{4\ A}$.

$k = -(2.7)(9.8)/(-0.035) = \underline{756\ \text{N/m}}$. $f = [756/2.7]^{\frac{1}{2}}/(2\pi) = \underline{2.66\ \text{Hz}}$

5.



t	0	$T/4$	$T/2$	$3T/4$	T	$5T/4$
x	A	0	$-A$	0	A	0

As $x = A$ when $t = 0$ this indicates
$x = A\cos(\omega t)$ or $x = A\cos(2\pi t/T)$; i.e., a cosine.

6. (a) $k = m(2\pi f)^2 = (0.0008)[(2\pi)(10)]^2 = \underline{3.16\ \text{N/m}}$.

(b) $k = m_1(2\pi f_1)^2 = m_2(2\pi f_2)^2$.

$f_2 = f_1(m_1/m_2)^{\frac{1}{2}} = 10(0.0008/0.0001)^{\frac{1}{2}} = \underline{5}$

7. $f_2 = f_1(m_1/m_2)^{\frac{1}{2}} = (2.5)(0.4/0.3)^{\frac{1}{2}} = \underline{2.9\ \text{Hz}}$

8. (a) $V_{max} = \omega A = 2\pi fA = (2\pi)(2)(0.18) = \underline{2.26\ m/s}$.

(b) $V = (V_{max}/A)(A^2 - x^2)^{\frac{1}{2}} = (2.26)[1 - (0.1/0.18)^2]^{\frac{1}{2}} = \underline{1.88\ m/s}$.

(c) Consider energy when mass passes equilibrium point. Velocity is maximum; potential energy is zero. Energy is $mV_{max}^2/2 = (0.26)(2.26)^2/2$
$= \underline{0.665\ J}$.

(d) $x = A \cos 2\pi ft$, i.e. $x = \underline{0.18 \cos (4\pi t)}$.

9. Frequency is inversely proportional to square root of mass, i.e. $m_2 = m_1 f_1^2/f_2^2$. Thus $m_1 + 0.7 = m_1(0.72/0.48)^2$. Solving, $m_1 = \underline{0.56\ kg}$.

10. $F = -2kx$. $f = (2\ k/m)^{\frac{1}{2}}/2\pi$

11. (a) Max x, at $A = 0$, is $A = \underline{0.32\ m}$

(b) $f = \dfrac{\omega}{2\pi} = (7.40\ s^{-1})/2\pi = \underline{1.18\ Hz}$

(c) $v_{max} = \omega A = (7.40\ s^{-1})(0.32\ m) = 2.37\ m/s$

$E = \frac{1}{2}mv_{max}^2 = (0.5)(0.50\ kg)(2.37\ m/s)^2 = \underline{1.40\ J}$

(d) $v = v_{max}(1 - x^2/A^2)^{\frac{1}{2}} = (2.37\ m/s)[1 - (0.26\ m)^2/(0.32\ m)^2]^{\frac{1}{2}} = 1.38\ m/s$
$KE = (0.5)(0.5\ kg)(1.38\ m/s)^2 = \underline{0.48\ J}$
$PE = E_{TOT} - KE = 1.40\ J - 0.48\ J = \underline{0.92\ J}$

12. $k = (1.80\ kg)(9.8\ m/s^2)/(0.275\ m) = 64.1\ N/m$

$T/4 = 2\pi(m/k)^{\frac{1}{2}}/4 = 2\pi[(1.80\ kg)/(64.1\ N/m]^{\frac{1}{2}}/4 = \underline{0.263\ s}$

13. $k = 80\ N/(0.20\ m) = 400\ N/m$

Energy stored $= \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$v = [(400\ N/m)(0.20\ m)^2/(0.150\ kg)]^{\frac{1}{2}} = \underline{10.3\ m/s}$

(a) $T = 2\pi(m/k)^{\frac{1}{2}} = 2\pi[(0.75\text{ kg})/(124\text{ N/m})]^{\frac{1}{2}} = 0.489\text{ s}$
$f = 1/T = \underline{2.05\text{ Hz}}$

(b) $v_{max} = \omega A$
$\omega = 2\pi(2.05\text{ Hz}) = 12.86\text{ rad/s}$
$A = (2.76\text{ m/s})/(12.86\text{ rad/s}) = \underline{0.215\text{ m}}$

(c) $a_{max} = \omega^2 A = (12.86\text{ rad/s})^2(0.215\text{ m}) = \underline{35.5\text{ m/s}^2}$

(d) $\underline{x = 0.215\sin(12.86\ t)}$

(e) $E = mv_{max}^2/2 = (0.5)(0.750\text{ kg})(2.76\text{ m/s})^2 = \underline{2.86\text{ J}}$

15. Most general simple harmonic motion is $x = x_0 \sin(\omega t + \alpha)$ where $\omega = 2\pi f = 2\pi/T$. If $x = 0$ when $t = 0$, $\alpha = 0$. Thus $x = x_0 \sin(2\pi t/T)$.

16. $x = x_0 \cos \omega t$. $v = v_{max} \sin \omega t$. $(v/v_{max})^2 + (x/x_0)^2 = 1$.

(a) $x = x_0[1 - 1/4]^{\frac{1}{2}} = \underline{x_0(3)^{\frac{1}{2}}/2}$.

(b) $a = -(k/m)x$. Maximum acceleration occurs at x_0. Acceleration is half at $x = \underline{x_0/2}$.

17. $\omega = (k/m)^{\frac{1}{2}} = [160/0.38]^{\frac{1}{2}} = 20.5\text{ s}^{-1}$.

(a) Hence, $\underline{x = 0.285\sin 20.5t}$.

(b) $t = T/4$ or $2\pi/4\omega = \underline{0.0766\text{ s}}$.
A hanging spring would have its minimum extension when compressed, which would be at

$\frac{3T}{4} = .230$ s and these would be repeated every T = .306 s.

18. Energy stored in vibration is $kx^2/2 = (6.8 \times 10^3)(0.174)^2/2 = 103\text{ J}$.

Hence $v_{max} = (2E/m)^{\frac{1}{2}} = [2(103)/0.412]^{\frac{1}{2}} = 22.4\text{ m/s}$.
By conservation of momentum, $v = (22.4)(0.412)/0.012 = \underline{767\text{ m/s}}$.

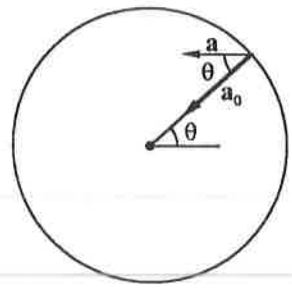
19. $E = kA^2/2$. $\omega = (k/m)^{\frac{1}{2}}$. $E = mA^2\omega^2/2$. $A_2 = A_1(E_2/E_1)^{\frac{1}{2}} = A_1(10)^{\frac{1}{2}}$. $\underline{A_1 = \sqrt{10}\,A_2}$

20. For equilibrium $mg = kx_0$. Hence extension is mg/k. If spring is compressed a distance x, the restoring force is $-k(x - x_0) - mg = -kx$
This has direction opposite to the displacement from equilibrium.

21. (a) We have to resolve the velocity "horizontally." Hence $v = -v_0 \sin \omega t$, i.e. towards the center.

(b) We have to resolve the acceleration "horizontally."
$\theta = \omega t \quad a = -a_0 \cos \theta = -a_0 \cos \omega t$

$$v_0 = \frac{2\pi}{T} x_0 = \omega x_0 = \sqrt{\frac{k}{m}}\, x \qquad a_0 = \frac{v_0^2}{x_0} = \frac{\omega^2 x_0^2}{x_0} = \frac{k}{m} x_0$$



22. (a) $F_1 = -k_1 x_1$, $F_2 = -k_2 x_2 = F_1 = F$.
$x = x_1 + x_2 = -F/k_1 - F/k_2 = -F(1/k_1 + 1/k_2)$
$F = -(1/k_1 + 1/k_2)^{-1} x$.
$T = 2\pi\sqrt{m/(1/k_1 + 1/k_2)^{-1}} = 2\pi\sqrt{m(1/k_1 + 1/k_2)}$.

(b) $x = x_2 = x_1$.
$F = -k_1 x - k_2 x = -(k_1 + k_2)x$.
$T = 2\pi\sqrt{m/(k_1 + k_2)}$.

23. (a) Period, time to make one vibration, is 60/44 = <u>1.36 s</u>.

(b) Frequency, number of vibrations/s, is 44/60 = <u>0.73 Hz</u>

24. $1\text{ s} = T = 2\pi(\ell/g)^{\frac{1}{2}}$. Hence $\ell = (9.8\text{ m/s})/(2\pi)^2 =$ <u>0.248 m</u>.

25. (a) $T = 2\pi(\ell/g)^{\frac{1}{2}} = 2\pi(0.8/9.8)^{\frac{1}{2}} = 1.80$ s.

(b) As g is zero, period is infinite.

26. (a) $\theta = \theta_0 \cos \omega t = \theta_0 \cos 2\pi f t = (15^\circ)\cos[2\pi(2)(0.25)] =$ <u>-15°</u>.

(b) $(15^\circ)\cos[2\pi(2)(1.6)] = 4.64^\circ$.

(c) $(15^\circ)\cos[2\pi(2)(500)] =$ <u>15°</u>. Note that the motion is only accurately harmonic for sma displacements.

(a) $f = (g/\ell)^{\frac{1}{2}}/2\pi = [(9.8\ \text{m/s}^2)/(0.36\ \text{m})]^{\frac{1}{2}}/2\pi = 0.830$ Hz

(b) $A = (0.36\ \text{m})(15^\circ)(2\pi/360^\circ) = 0.0942$ m
$v_{max} = \omega A = 2\pi fA = 2\pi(0.830\ \text{Hz})(0.0942\ \text{m}) = \underline{0.49\ \text{m/s}}$

28. Conservation of energy gives $mgh = mv^2/2$. Hence $v_0 = [2\,gL(1 - \cos\theta_0)]^{\frac{1}{2}}$.
If we measure θ_0 in radians and assume it is small $\cos\theta_0 = 1 - \theta_0^2/2$.

Hence $v_0 = \theta_0(gL)^{\frac{1}{2}}$. An alternative derivation is $\Omega = (\Delta\theta_0/\Delta t)_{max} = \omega\theta_0 = \theta_0(g/L)^{\frac{1}{2}}$.

But $v_0 = \Omega L = \underline{\theta_0(gL)^{\frac{1}{2}}}$.

29. $v = f\lambda = \lambda/T = 12/5 = \underline{2.4\ \text{m/s}}$

30. $\lambda = v/f = (330\ \text{m/s})/(262\ \text{Hz}) = \underline{1.26\ \text{m}}$.

31. $\lambda = (3 \times 10^8\ \text{m/s})/(550 \times 10^3\ \text{Hz}) = \underline{545\ \text{m}}$. $\lambda = (3 \times 10^8\ \text{m/s})/(1600 \times 10^3\ \text{Hz}) = \underline{188\ \text{m}}$.
$\lambda = (3 \times 10^8\ \text{m/s})/(88 \times 10^6\ \text{Hz}) = \underline{3.41\ \text{m}}$. $\lambda = (3 \times 10^8\ \text{m/s})/(108 \times 10^6\ \text{Hz}) = \underline{2.78\ \text{m}}$.

32. (a) $v = (B/\rho)^{\frac{1}{2}} = (2 \times 10^9\ \text{N/m}^2/1000\ \text{kg/m}^3)^{\frac{1}{2}} = \underline{1414\ \text{m/s}}$.

(b) $v = (E/\rho)^{\frac{1}{2}} = (45 \times 10^9\ \text{N/m}^2/2.7 \times 10^3\ \text{kg/m}^3)^{\frac{1}{2}} = \underline{4082\ \text{m/s}}$.

33. $v = (B/\rho)^{\frac{1}{2}}$. Hence speed is greater in rod with smallest density by a factor $(2)^{\frac{1}{2}} = \underline{1.41}$.

34. $v = (F_T L/m)^{\frac{1}{2}} = [(150)(30)/0.55]^{\frac{1}{2}} = 90.5$ m/s.
$t = L/v = 30/90.5 = \underline{0.332\ \text{s}}$.

35. $\lambda = v/f = (E/\rho)^{\frac{1}{2}}/f = (100 \times 10^9\ \text{N/m}^2/7.8 \times 10^3\ \text{kg/m}^3)^{\frac{1}{2}}/10^4\ \text{Hz} = \underline{0.36\ \text{m}}$.

36. $d = tv/2 = (t/2)(B/\rho)^{\frac{1}{2}} = (2/2)(2 \times 10^9/1000) = \underline{1410\ \text{m}}$.

37. (a) $t_s = d/v_s$. $t_p = d/t_p$. Thus $t_s - t_p = d(1/v_s - 1/v_p)$.
Hence $d = (1.8 \times 60)/[1/5000 - 1/9000] = 1.215 \times 10^6$ m, or $\underline{1200\ \text{km}}$.

(b) The quake could be anywhere on a circle of radius d. So, <u>no</u>.

38. $E = 2\pi^2 m f^2 x_0^2$. Thus $x_2 = x_1(E_2/E_1)^{\frac{1}{2}} = x_1(2)^{\frac{1}{2}} = \underline{1.41\ x_1}$.

39. (a) Displacement is $x_0 - (-x_0) = 2x_0$, i.e., $x_0 = 0.16/2 = \underline{0.08\ \text{m}}$.

(b) $E_2 = E_1(x_2/x_1)^2 = E_1(0.12/0.08)^2 = \underline{2.25\ E_1}$.

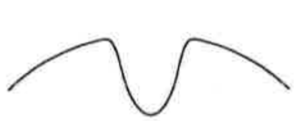
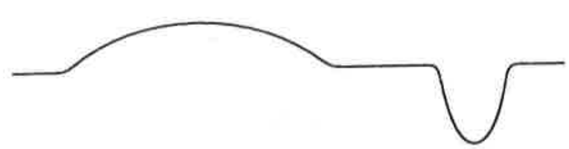
40. (a) $I_2 = I_1(d_1^2/d_2^2) = I_1(10\ \text{km})^2/(20\ \text{km})^2 = \underline{0.25\ I_1}$.

(b) $x_2 = x_1(d_1/d_2) = x_1(10\ \text{km}/20\ \text{km}) = \underline{0.5\ x_1}$.

41. (a) $I_2 = I_1(d_1^2/d_2^2) = (1.0 \times 10^6\ \text{J/m}^2\cdot\text{s})(100\ \text{km})^2/(1\ \text{km})^2 = \underline{10^{10}\ \text{J/m}^2\cdot\text{s}}$.

(b) Energy/s $= \underline{5 \times 10^{10}\text{W}}$.

42. The intensity is the energy/s flowing through each meter of circumference. As total energy P conserved $I = P/2\pi r(\text{J/m}\cdot\text{s})$. But the amplitude is proportional to $(I)^{\frac{1}{2}}$. Hence $x_0 \propto 1/(r)^{\frac{1}{2}}$.

43. (a)  (b) 

(c) <u>It is stored in kinetic energy</u> of the string.

44. $v_2 = v_1 \sin\theta_2/\sin\theta_1 = (9\ \text{km/s})\sin 27°/\sin 47° = \underline{5.59\ \text{km/s}}$

45. $\sin\theta_2 = \sin\theta_1(v_2/v_1) = \sin 35°(2.1/2.6) = 0.463$. Hence, $\theta_2 = 27.6° = \underline{28°}$ to shelf..

46. $\sin\theta_2 = \sin\theta_1(v_2/v_1) = \sin\theta_1(\rho_1/\rho_2)^{\frac{1}{2}} = (\sin 20°)(3.8/2.7)^{\frac{1}{2}} = 0.406$. Hence $\theta_2 = \underline{23.9°}$.

47. (a) The refracted $\theta_2 = 90°$ (not $\theta_1 = 90°$)

$$\sin\theta_{in} = \sin\theta_2\frac{v_1}{v_2} = \sin 90°\frac{v_1}{v_2} = \frac{v_1}{v_2}$$

$$\sin\theta_{in} = \frac{v_1}{v_2} \text{ so } \theta_{in} = \text{arc sin}\left[\frac{v_1}{v_2}\right]$$

$$\underline{\theta_{in}} = \theta_2 = \underline{\text{arc sin}(v_1/v_2)}$$

(b) $\underline{\theta_{in}} = \text{arc sin}[(7.5\text{ km/s})/(9.3\text{ km/s})] = 53.8°$

No transmission when $\underline{\theta \geq 54°}$.

48. $f_n = nf_1$, n = 1, 2, 3, · · ·
$f_1 = \underline{294\text{ Hz}}$
$f_2 = \underline{588\text{ Hz}}$
$f_3 = \underline{882\text{ Hz}}$
$f_4 = \underline{1176\text{ Hz}}$

Velocity on string remains the same. First $\lambda_1 = L/2$, second $\lambda_2 = (3/4)L/2$.
Hence $f_2 = f_1(\lambda_1/\lambda_2) = 440(4/3) = \underline{587\text{ Hz}}$

50. $f_4 = 4f_1 = 360$ Hz
$f_1 = \underline{90\text{ Hz}}$, $f_2 = \underline{180\text{ Hz}}$, $f_3 = \underline{270\text{ Hz}}$

51. Nodes are $\lambda/2$ apart
$d = v/2f = (350\text{ m/s})/2(420\text{ Hz}) = \underline{0.417\text{ m}}$

52. $f_n = nf_1 = 360$ Hz
$f_{n+1} = (n + 1)f_1 = 400$ Hz
So $f_1 = \underline{40\text{ Hz}}$

53. $v = [F_T/(m/L)]^{\frac{1}{2}} = [(520\text{ N})(0.9\text{ m})/(3.6 \times 10^{-3}\text{ kg})]^{\frac{1}{2}} = 361$ m/s. The fundamental has $\lambda/2 = 0.6$ m.
Hence $f = v/\lambda = (361\text{ m/s})/(1.2\text{ m}) = \underline{301\text{ Hz}}$. Over-tones are $\underline{602\text{ Hz}}$, $\underline{903\text{ Hz}}$.

54. The velocity increases as the square root of the tension and therefore so does the frequency.

Hence $f_2 = f_1(T_2/T_1)^{\frac{1}{2}} = (294\text{ Hz})(1.10)^{\frac{1}{2}} = \underline{308\text{ Hz}}$.

55. $f_1 = v/\lambda = [(F_T/\mu)]^{\frac{1}{2}}/2\ L$. For each harmonic $f_n = nf_1$.

56. $F_T = mg$
Use result from Problem 55.

$$f = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$F_T = 4f^2L^2/\mu n^2$
$m = 4f^2L^2/\mu g n^2$

(a) $m_1 = 4(60\text{ Hz})^2(1.40\text{ m})^2(4.2 \times 10^{-4}\text{ kg/m})/[(9.8\text{ m/s}^2)(1)^2] = \underline{1.21\text{ kg}} = \underline{1.2\text{ kg}}$

(b) $m_N = m_1/n^2$
$m_2 = 1.21\text{ kg}/(2)^2 = 0.302\text{ kg} = \underline{0.30\text{ kg}}$

(c) $m_5 = 1.21\text{ kg}/(5)^2 = \underline{0.048\text{ kg}}$

The amplitude of standing wave can be much greater since the vibrator is near a node and th displacement at an antinode is much larger.

57. $$v = \sqrt{\frac{T}{\rho}} \Rightarrow f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

$T \Rightarrow$ tension $\qquad L \Rightarrow$ length
$\rho \Rightarrow$ linear density

$$f + \Delta f = \frac{1}{2L}\sqrt{\frac{T + \Delta T}{\rho}} = \frac{1}{2L\sqrt{\rho}}(T + \Delta T)^{\frac{1}{2}}.$$

$$f + \Delta f = \frac{\sqrt{T}}{2L\sqrt{\rho}}\left[\frac{T + \Delta T}{T}\right]^{\frac{1}{2}} = \frac{1}{2L}\sqrt{\frac{T}{\rho}}\left[1 + \frac{\Delta T}{T}\right]^{\frac{1}{2}}$$

$$f + \Delta f = f\left[1 + \frac{\Delta T}{2T} + \cdot\cdot\cdot\right]$$

$$f + \Delta f = f + \frac{\Delta T}{2T}f + \cdot\cdot\cdot$$

$$\Delta f = \left[\frac{\Delta T}{2T}\right]f.$$

For $\Delta f = 4$ Hz $\Rightarrow 4\text{ Hz} = \left[\frac{\Delta T}{2T}\right]438\text{ Hz}$

$$\frac{\Delta T}{T} = \frac{8}{438} \times 100 = 1.8\%.$$

General formula $f_n = \frac{n}{2L}\sqrt{\frac{T}{\rho}}$.

n being an integer does not affect the analysis. Hence the formula should apply to harmonics.

58. Here we have a wave open at both ends of tub with a node in the middle. Hence $(\lambda/2) = L$. Thus $v = f\lambda = (0.60\text{ Hz})(1\text{ m}) = \underline{0.60\text{ m/s}}$.

59. From conservation of energy $mgh = kx^2/2$ for intial extension, where h is total height dropped, i.e., 16.2 m. When lying in it $mg = kX$. Hence $X = x^2/2h = (1.2\text{ m})^2/2/16.2\text{ m} = \underline{0.044\text{ m}}$.
For a 30 m jump $mg(30 + X) = kX^2/2$. Also $mg(16.2) = k(1.2\text{ m})^2/2$.
Divide and solve for X gives $\underline{1.7\text{ m}}$.

60. (a) Acceleration upwards increases g to $(3/2)g$. But $f = 2\pi(g/\ell)^{\frac{1}{2}}$.

Hence $f_2 = f_1(g_2/g_1)^{\frac{1}{2}} = f(3/2)^{\frac{1}{2}} = \underline{1.22\ f}$.

(b) Accleration downward decreases g to $g/2$. Hence $f_2 = f(1/2)^{\frac{1}{2}} = \underline{0.7071\ f}$.

61. $kx = mg$
$k = (75\text{ kg})(9.8\text{ m/s}^2)/(0.045\text{ m}) = 1.633 \times 10^4\text{ N/m}$

(a) $f = (k/m)^{\frac{1}{2}}/2\pi = [(1.633 \times 10^4\text{ N/m})/(400\text{ kg})]^{\frac{1}{2}}/2\pi = \underline{1.02\text{ Hz}}$

(b) Ignore kinetic energy stored in the water.
$v_{max} = 2\pi fA = 2\pi(1.02\text{ Hz})(0.045\text{ m}) = 0.287\text{ m/s}$

$E = \frac{1}{2}mv_{max}^2 = (0.5)(400\text{ ks})(0.287\text{ m/s})^2 = \underline{16.5\text{ J}}$

62. $f = v/\lambda = 2\pi fr/\lambda = (2\pi)(0.112\text{ m})(33/60\text{ Hz})/(2.1 \times 10^{-3}\text{ m}) = \underline{184\text{ Hz}}$.
Note that 33 rpm records are actually designed for 33 1/3 rpm.

63. (a) For G, first two overtones are $2(392\text{ Hz}) = \underline{784\text{ Hz}}$, $3(392\text{ Hz}) = \underline{1176\text{ Hz}}$
For A, overtones are $\underline{880\text{ Hz}}$, $\underline{1320\text{ Hz}}$.

(b) $f = (h/2L)(F_T/\mu)^{\frac{1}{2}} = (h/2)(F_T/mL)^{\frac{1}{2}}$
$M_G/M_A = F_A^2/F_G^2 = (440/392)^2 = \underline{1.26}$

(c) Use $f = (h/2L)(F_T/\mu)^{\frac{1}{2}}$
$L_G/L_A = f_A/f_G = (440/392) = \underline{1.12}$

(d) $F_{TG}/F_{TA} = f_G^2/f_A^2 = (392/440)^2 = 0.794$

64. (a) $kx^2/2 = mv^2/2$. Thus $x = v(m/k)^{\frac{1}{2}} = (2.5)(5/85)^{\frac{1}{2}} = \underline{0.606\ m}$

(b) $2\pi f = (k/m)^{\frac{1}{2}}$. Thus $t = T/2 = 1/(2f) = \pi(m/k)^{\frac{1}{2}} = \pi(5/85)^{\frac{1}{2}} = \underline{0.762\ s}$.

65. Period = 25 h/2 = $\underline{12.5\ h}$.
Frequency = 1/12.5 h = 1/(12.5 × 3600) = $\underline{2.2 \times 10^{-5}\ Hz}$.
Velocity = λf = (20000 × 10^3)(2.2 × 10^{-5}) = $\underline{440\ m/s}$.

66. (a) $v_2/v_1 = (\mu_1/\mu_2)^{\frac{1}{2}}$ where μ_1 and μ_2 are the masses per unit lengths of the two strings. The tension in the strings will be the same.

(b) The frequency is the same if the "joining" ends of the two strings vibrate together.

Hence $\lambda_2/\lambda_1 = (v_2/f)/(v_1/f) = (\mu_1/\mu_2)^{\frac{1}{2}}$.

(c) The wave length is inversely proportional to the mass, m, i.e. is shorter in heavier cord.

67. $a_{max} = 4\pi^2 f^2 A < g$. $A < g/(4\pi^2 f^2) = 9.8/[4\pi^2(36)] = 0.0069$ m, or $\underline{6.9\ mm}$.

68. Restoring force is weight of water displaced = $mg = \rho V g = \rho g A \Delta x$. So $F = -\rho g A \Delta x$.
Hence $k = \underline{\rho g A}$ and we have SHM.

69. $\theta = \theta_0 \sin \omega t$. Hence $t_5 - t_{-5} = (2/\omega)$ arc sin $(\theta/\theta_0) = (1.05/\omega) = T(1.05)(2\pi) = 0.167\ T$.
But it is also between 5° and −5° on the swing back so total fraction is $\underline{0.334}$.

70. $F = kx - mg$

At $x = 15$ cm, $F = 0$ N

$k(0.15\text{ m}) - m(9.8\text{ m/s}^2) = 0$
$k/m = (9.8\text{ m/s}^2)/(0.15\text{ m}) = 65.33\ s^{-1}$

$\omega = \sqrt{k/m} = 8.08$ rad/s
$f = \omega/2\pi = \underline{1.29\ Hz}$

or could just use conservation of energy, $mgh = \frac{1}{2}kh^2$

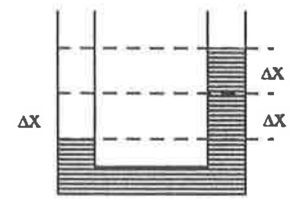
$\frac{k}{m} = \frac{2g}{h} = 2(9.8)/0.3 = 65.33\ s^{-1}$

71. As length increases the period will increase; hence, clock will be slow.
New length $\ell' = \ell + \Delta\ell = \ell(1 + \alpha\Delta T)$
$\alpha = 19 \times 10^{-6}{}^{\circ}C^{-1}$,

$$f = \frac{1}{2\pi}[g/\ell(1 + \alpha\Delta T)]^{\frac{1}{2}} = f_0(1 - \alpha\Delta T/2)$$

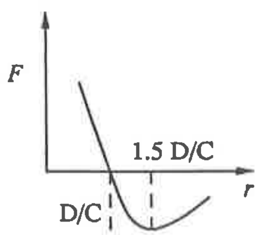
In 24 hrs, number of vibrations is
$(8.64 \times 10^4\ s)\ f_0\ (1 - \alpha\Delta T/2)$
Clock is designed so that it reads 8.64×10^4 s have passed in $(8.64 \times 10^4\ s)\ f_0$ vibrations.
Hence, it now reads that $(8.64 \times 10^4\ s)f_0(1 - \alpha\Delta T/2)$ time has passed.
$f_0 = 1/0.4820$ s, $\alpha = 19 \times 10^{-6}{}^{\circ}C^{-1}$, $\Delta T = 20^{\circ}$
$\Delta t = -(8.64 \times 10^9 s)(1/0.4820)(19 \times 10^{-6}{}^{\circ}C^{-1})(20^{\circ}C)/2 = -34.1$ s, or 3.41 sec lost.

72.



The excess weight displaced water is $\rho ghA = 2\rho g\Delta xA$. This is the resultant restoring force which if we identify with $k\Delta x$ gives $k = 2\ \rho gA$ and the motion is harmonic.

73. (a)



(b) $F = 0$ when $r = D/C$.

(c) Use binomial theorem.
$r^{-2} = (r_0 + \Delta r)^{-2} \simeq r_0^{-2} - 2\Delta r/r_0^3$.
$r^{-3} = (r_0 + \Delta r)^{-3} \simeq r_0^{-3} - 3\Delta r/r_0^4$.
But $F = +C[-r^{-2} + r_0 r^{-3}] \simeq -(C/r_0^3)\Delta r$, i.e. SHM with

(d) $k = \dfrac{C^4}{D^3}$

(e) $T = 2\pi\sqrt{\dfrac{MD^3}{C^4}}$

Chapter 12

1. $d = (vt)/2 = (343 \text{ m/s})(1.20 \text{ s})/2 = \underline{206 \text{ m}}$.

2. (a) $\lambda = v/f = (1440 \text{ m/s})(250{,}000 \text{ Hz}) = \underline{5.76 \times 10^{-3} \text{ m}}$.

   (b) $\lambda = (343 \text{ m/s})/(250{,}000 \text{ Hz}) = \underline{1.37 \times 10^{-3} \text{ m}}$.

3. The velocity of sound in concrete is $(E/\rho)^{\frac{1}{2}} = (2 \times 10^{10} \text{ N/m}^2/2.3 \times 10^3 \text{ kg/m}^3)^{\frac{1}{2}} = 2949 \text{ m/s}$.
Time through concrete is $(d)/(2949 \text{ m/s})$. Time through air is $(d)/(343 \text{ m/s})$.
Hence $d = (1.2 \text{ s})/[1/(343 \text{ m/s}) - 1/(2949 \text{ m/s})] = 470 \text{ m} = \underline{4.7 \times 10^2 \text{ m}}$.

4. $\beta = 10 \log[10^{-7}/10^{-12}] = \underline{50 \text{ dB}}$

5. $60 = 10 \log [I_0/10^{-12}]$. $\log I_0 = -6$. $I_0 = \underline{10^{-6} \text{ W/m}^2}$.

6. $1 \text{ dB} = 10 \log I_2/I_1$. $I_2/I_1 = 10^{0.1} = 1.26$. The amplitude ratio is the square root of this, i.e. 1.12.

7. $62 \text{ dB} = 10 \log(I_2/I_1)$. $I_2/I_1 = 10^{6.2} = \underline{1.58 \times 10^6}$.

8. $95 \text{ dB} = 10 \log[2\, I/(10^{-12} \text{ W/m}^2)]$. Thus $10 \log I/(10^{-12} \text{ W/m}^2) = (95 - 10 \log 2)\text{dB} = \underline{92.0 \text{ dB}}$.

9. $95 = 10 \log [I/10^{-12}]$. $I = 10^{-2.5} \text{ W/m}^2 = 3.16 \times 10^{-3} \text{ W/m}^2$.
Energy/sec $= (3.16 \times 10^{-3})(5 \times 10^{-5}) = \underline{1.6 \times 10^{-7} \text{ W}}$.

10. (a) $IA = (3 \times 10^{-6} \text{ W/m}^2)(2\pi)(0.5 \text{ m})^2 = \underline{4.71 \times 10^{-6} \text{ W}}$.

    (b) Number to produce 100 W is $(100 \text{ W})/(4.71 \times 10^{-6} \text{ W}) = \underline{2.12 \times 10^7}$.

11. $80 \text{ dB} = 10 \log[I_1/(10^{-12} \text{ W/m}^2)]$; $85 \text{ dB} = 10 \log[I_2/(10^{-12} \text{ W/m}^2)]$.
$I_1 + I_2 = (10^{-12} \text{ W/m}^2)[10^8 + 10^{8.5}] = 4.16 \times 10^{-4} \text{ W/m}^2$.
Hence $\beta = 10 \log (4.16 \times 10^{-4}/10^{-12}) = \underline{86.2 \text{ dB}}$.